On the harmonic flow of geometric structures

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Abstract

In this talk, I will report on recent results of an ongoing collaboration with Éric Loubeau, Andrés Moreno and Henrique Sa Earp on the study of the harmonic flow of -structures. This is the negative gradient flow of a natural Dirichlet-type energy functional on an isometric class of -structures on a closed Riemannian -manifold, where is the stabilizer in of a finite collection of tensors in . Using general Bianchi-type identities of -structures, we are able to prove monotonicity formulas for scale-invariant local versions of the energy, similar to the classic formulas proved by Struwe and Chen (1988-89) in the theory of harmonic map heat flow. We then deduce a general epsilon-regularity result along the harmonic flow and, more importantly, we get long-time existence and finite-time singularity results in parallel to the classical results proved by Chen-Ding (1990) in harmonic map theory. In particular, we show that if the energy of the initial -structure is small enough, depending on the -norm of its torsion, then the harmonic flow exists for all time and converges to a torsion-free -structure. Moreover, we prove that the harmonic flow of -structures develops a finite time singularity if the initial energy is sufficiently small but there is no torsion-free -structure in the homotopy class of the initial -structure. Finally, based on the analogous work of He-Li (2021) for almost complex structures, we give a general construction of examples where the later finite-time singularity result applies on the flat -torus, provided the -th homotopy group of the quotient is non-trivial; e.g. when and , or when and .