SU(3)-structures of LT type

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Based on:

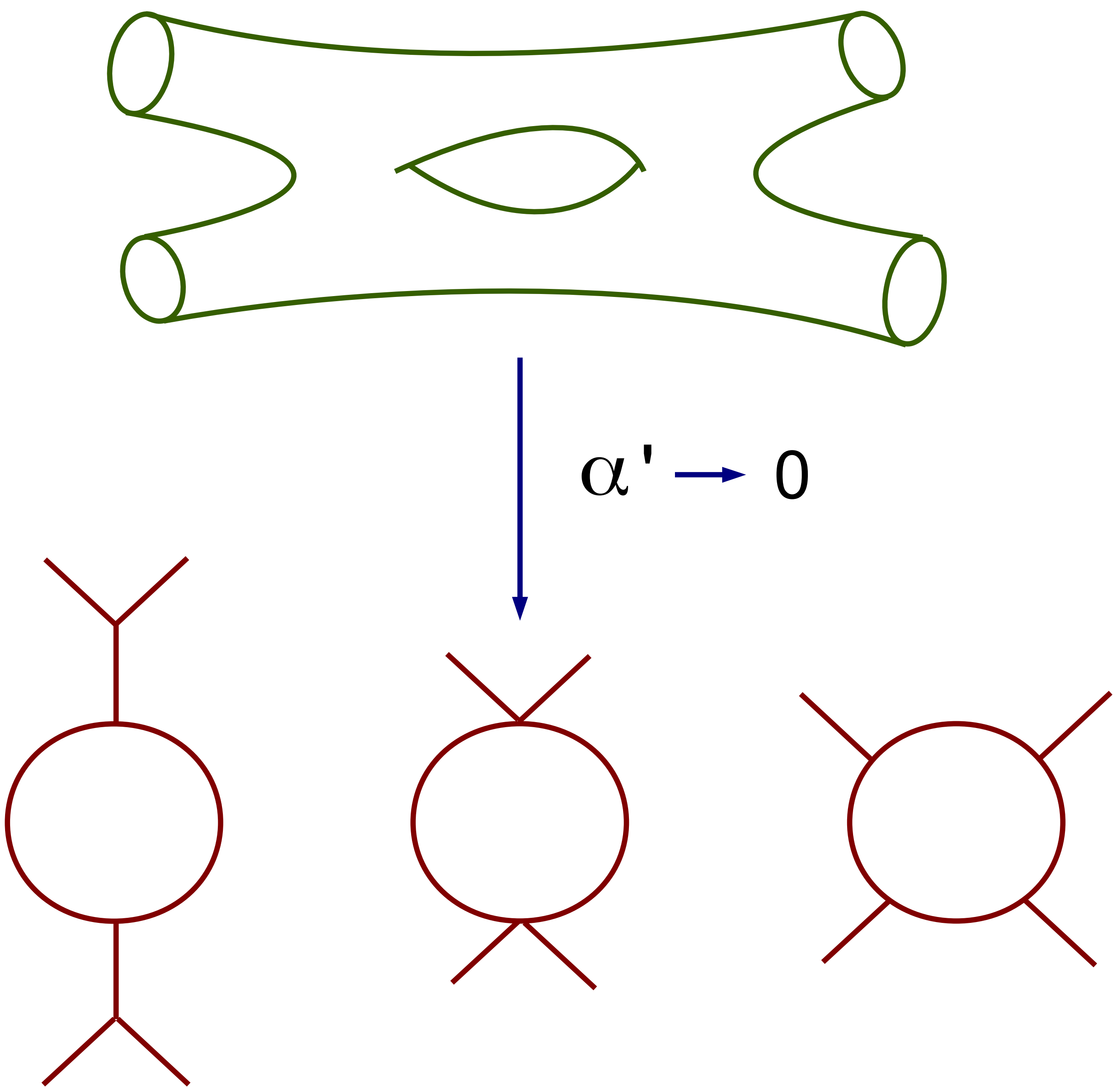
* *D. Lüst & DT*, hep-th/0412250 (2004)
* *P. Koerber & DT*, 0804. 0614 (2008)
* *M. Larfors, D. Lüst & DT*, 1005.2194 (2010)
* *DT*, 1206.5900 (2012)
* *R. Terrisse & DT*, 1707.04636 (2017)
* *M. Larfors, A. Lukas, F. Ruehle & DT*, 2022

Outline

- Introduction: context & motivation
- Characterization of LT solutions
- Constructions of LT solutions
- Conclusions: outlook & wishlist

Introduction

- String/M-theory is a leading candidate for a consistent theory of quantum gravity.
- At low energies (critical) string theory reduces to 10d supergravity



Introduction

- A great part of our knowledge of the theory (dualities, holography, branes, black holes,...) comes from the study of its supersymmetric, (bosonic) supergravity vacua (solutions)
- 10d supergravity contains (in addition to the graviton) fermions and higher-rank antisymmetric tensors (flux).

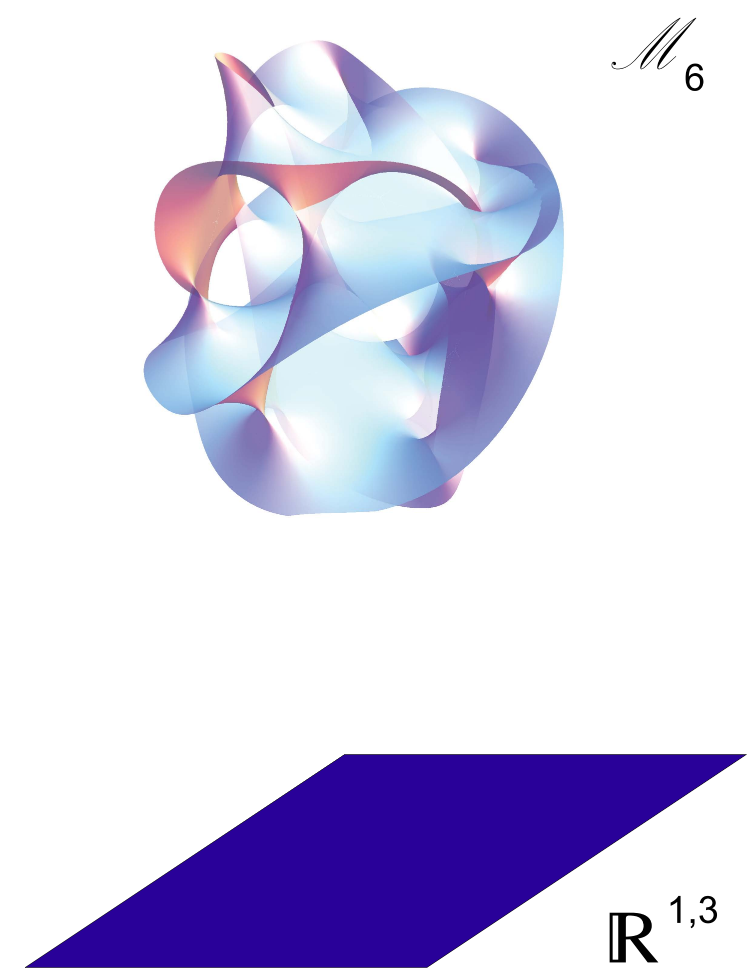
Introduction

- In the absence of flux, *susy* vacua of the theory are of the form $\mathbb{R}^{1,3} \times \mathcal{M}_6$ where $\mathcal{M}_6$ is a Calabi-Yau manifold.
- Good control of the physics, beautiful mathematics.
- Powerful tools from *math.AG*.

* *Candelas, Horowitz, Strominger, Witten*, 1985

* *Strominger, Witten*, 1985

* *http://hep.itp.tuwien.ac.at/-kreuzer/CY/*



Introduction

- Good physical reasons to turn on the flux (generic case).
- A famous no-go theorem excludes Minkowski *flux* vacua, provided:
  - absence of sources, no (or mild) singularities
  - compactness
  - two-derivative actions
  - the *Strong Energy Condition* is obeyed by the 10d/11d theory

* *Gibbons*, 1984
* *Maldacena & Nuñez*, 2000

Introduction

- In the presence of flux: ***susy*** vacua of the form $\mathrm{AdS}_4 \times \mathcal{M}_6$ where $\mathcal{M}_6$ is ***not*** special-holonomy.

- Holography, moduli stabilisation, de Sitter uplifts, ...

* *Freund, Rubin*, 1980
* *Duff, Pope*, 1982
* *Nilsson, Pope*, 1984

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* *Freund, Rubin*, 1980
* *Duff, Pope*, 1982
* *Nilsson, Pope*, 1984

- G-structures, generalized (complex) geometry, ...

* *Gauntlett, Kim, Martelli, Waldram*, 2001
* *Graña, Minasian, Petrini, Tomasiello*, 2004

Introduction

- LT-vacua are $\mathcal{N} = 1$ solutions of (massive) 10d IIA supergravity of the form $\mathrm{AdS}_4 \times \mathcal{M}_6$ with $\mathcal{M}_6$ in a certain subclass of *half-flat*.

* *D. Lüst & DT*, 2004

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- Holographically dual to 3d Chern-Simons theories

* *Gaiotto & Tomasiello*, 2010

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* *DeWolfe, Giryavets, Kachru, Taylor*, 2005

* *Acharya, Benini, Valandro*, 2007

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- Relevant to the question of *scale separation* and the *swampland*

  * *Many recent papers*

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IIA supergravity

- IIA (bosonic) action

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big(-R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2 + \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2 + \frac{1}{2}m^2 e^{5\phi/2}\Big) + S_{\mathrm{CS}}$$

- Bianchi identities

$$\mathrm{d}F = mH \;; \quad \mathrm{d}H = 0 \;; \quad \mathrm{d}G = H\wedge F$$

- Supersymmetry of the (bosonic) vacuum

$$\nabla_M\epsilon = \frac{1}{16}me^{5\phi/4}\Gamma_M\epsilon + \frac{1}{32}e^{3\phi/4}\Gamma_{MNP}F^{NP}\epsilon + \dots$$

$$0 = \Big(10me^{5\phi/4} - 3e^{3\phi/4}\Gamma_{MN}F^{MN} + \dots\Big)\epsilon$$

4d supersymmetric solutions

- An *SU(3)*-structure on $\mathcal{M}_6$
  - A complex decomposable 3-form Ω
  - A real 2-form J such that

$$\Omega \wedge J = 0 \; ; \qquad \Omega \wedge \Omega^* = \frac{4i}{3} J \wedge J \wedge J \neq 0$$

- Equivalences:
  - An *SU(3)*-structure on $\mathcal{M}_6$
  - Existence on $\mathcal{M}_6$ of $(g, \mathcal{I})$ with $c_1(\mathcal{I}) = 0$
  - Existence on $\mathcal{M}_6$ of (g, η) with η ***pure*** & nowhere-vanishing

4d supersymmetric solutions

- Link with supergravity:
  - *Susy* parameter: $\epsilon \sim \zeta \otimes \eta$
  - *SU(3)*-structure: $\Omega \sim \eta\gamma_{(3)}\eta \; ; \quad J \sim \eta^\dagger\gamma_{(2)}\eta$

- Torsion classes:

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}(W_1^*\Omega) + W_4 \wedge J + W_3$$

$$\mathrm{d}\Omega = W_1 J \wedge J + W_2 \wedge J + \Omega \wedge W_5^*$$

where:

$$W_1 \sim \mathbf{1} \oplus \mathbf{1}; \quad W_2 \sim \mathbf{8} \oplus \mathbf{8}; \quad W_3 \sim \mathbf{6} \oplus \bar{\mathbf{6}}; \quad W_4 \sim \mathbf{3} \oplus \bar{\mathbf{3}}; \quad W_5 \sim \mathbf{3}$$

4d supersymmetric solutions

- Link with supergravity:
  - *Susy* parameter: $\epsilon \sim \zeta \otimes \eta$
  - *SU(3)*-structure: $\Omega \sim \eta\gamma_{(3)}\eta$; $J \sim \eta^\dagger\gamma_{(2)}\eta$

- Torsion classes:

$$\nabla_m \eta = \frac{1}{2}\left(W_{4m}^{(1,0)} + W_{5m} - \text{c.c}\right)\eta$$

$$+\frac{1}{16}\left(4W_1 g_{mn} - 2W_4^p \Omega_{pmn} + 4iW_{2mn} - iW_{3mpq}\Omega^{pq}{}_n\right)\gamma^n \eta^c$$

where:

$$W_1 \sim \mathbf{1} \oplus \mathbf{1}; \quad W_2 \sim \mathbf{8} \oplus \mathbf{8}; \quad W_3 \sim \mathbf{6} \oplus \bar{\mathbf{6}}; \quad W_4 \sim \mathbf{3} \oplus \bar{\mathbf{3}}; \quad W_5 \sim \mathbf{3}$$

4d supersymmetric solutions

- Link with supergravity:
  - *Susy* equations:

$$\nabla\eta = F\cdot\gamma\eta$$

  - Compare with:

$$\nabla\eta = W\cdot\gamma\eta$$

- Conclusion:
  - The flux is expressed in terms of torsion classes: $F \sim W$
  - Constraints on the torsion classes.

4d supersymmetric solutions

- What about the equations of motion?

An *integrability theorem* guarantees that (potentially in the presence of *calibrated* sources) the Einstein, dilaton and 3-form eom's automatically follow from *susy*, the (generalized) *Bianchi identities* and a certain (mild) assumption on the form of the solution.

* *D. Lüst & DT*, 2004
* *P. Koerber & DT*, 2007
* *D. Lüst, F. Marchesano, L. Martucci & DT*, 2008
* *D. Prins & DT*, 2013

The LT solutions

- The 10d metric is of the form $\mathrm{AdS}_4 \times \mathcal{M}_6$
- An $SU(3)$-structure on $\mathcal{M}_6$ with torsion classes:

$$\mathrm{d}J = -\frac{3}{2}\, iW_1^- \mathrm{Re}\Omega \; ; \qquad \mathrm{d}\Omega = W_1^- J \wedge J + W_2^- \wedge J$$

- Fluxes:

$$H = \frac{2m}{5} e^{\phi} \mathrm{Re}\Omega$$

$$F_2 = \frac{1}{4} i e^{-\phi} W_1^- J + i e^{-\phi} W_2^-$$

$$F_4 = \frac{9}{4} i e^{-\phi} W_1^- \mathrm{vol}_4 + \frac{3m}{10} J \wedge J$$

$$\frac{1}{L^2} = \left(\frac{1}{25} m^2 + \frac{9}{16} |W_1^-|^2\right) e^{2\phi}$$

- Geometrical problem: construct $\mathcal{M}_6$ with this structure!

The LT solutions

- Generically there are D6/O6 sources:

$$\mathrm{d}F_2 + mH = j_6$$

$$j_6 \equiv \left(\frac{2m^2}{5}e^{\phi} - \frac{3}{8}|W_1^-|^2 e^{-\phi}\right)\mathrm{Re}\Omega + i e^{-\phi}\mathrm{d}W_2^-$$

- Absence of sources equivalent to:

$$\mathrm{d}W_2^- \propto \mathrm{Re}\Omega \;; \quad 3|W_1^-|^2 - |W_2^-|^2 \geq 0$$

The LT solutions

- The *Nearly Kähler* limit:

$$\mathrm{d}J = -\frac{3}{2}\, iW_1^- \mathrm{Re}\Omega \ ; \qquad \mathrm{d}\Omega = W_1^- J \wedge J$$

  - solutions without sources always possible
  - four homogeneous spaces: $S^3 \times S^3, \ \mathbb{CP}^3, \ \mathrm{Tw}(\mathbb{CP}^2), \ S^6$

* *K. Behrndt, M. Cvetic*, 2004

- The *Nearly Calabi-Yau* limit:

$$\mathrm{d}J = 0 \ ; \qquad \mathrm{d}\Omega = W_2^- \wedge J$$

  - solutions without sources never possible

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Explicit examples: cosets

- $\mathcal{M}_6$ of the form *G/H* with *H* subgroup of *SU(3)*

G	H
G_2	SU(3)
SU(3)×SU(2)^2	SU(3)
Sp(2)	S(U(2)×U(1))
SU(3)×U(1)^2	S(U(2)×U(1))
SU(2)^3×U(1)	S(U(2)×U(1))
SU(3)	U(1)×U(1)
SU(2)^2×U(1)^2	U(1)×U(1)
SU(3)×U(1)	SU(2)
SU(2)^3	SU(2)
SU(2)^2×U(1)	U(1)
SU(2)^2	1

* *D. Lüst, P. Koerber & DT*, 2008

Explicit examples: cosets

- Given a basis of the algebras and a coset representative L

$$\{\mathcal{H}_a\},\ a=1,\ldots,\dim(H)\ ;\quad \{\mathcal{K}_i\},\ i=1,\ldots,\dim(G)-\dim(H)$$

a coframe e^i on G/H is defined

$$L^{-1}dL = e^i\mathcal{K}_i + \omega^a\mathcal{H}_a$$

- A p-form $\phi = \frac{1}{p!}\phi_{i_1\ldots i_p} e^{i_1}\wedge\cdots\wedge e^{i_p}$ is *left-invariant* iff

$$f^j{}_{a[i_1}\phi_{i_2\ldots i_p]j} = 0\ ;\qquad \phi_{i_1\ldots i_p} = \text{const.}$$

where
$$[\mathcal{H}_a,\mathcal{H}_b] = f^c{}_{ab}\mathcal{H}_c\ ,$$
$$[\mathcal{H}_a,\mathcal{K}_i] = f^j{}_{ai}\mathcal{K}_j\ ,$$
$$[\mathcal{K}_i,\mathcal{K}_j] = f^k{}_{ij}\mathcal{K}_k + f^a{}_{ij}\mathcal{H}_a\ .$$

- The exterior differential of a left-invariant form is left-invariant

Explicit examples: cosets

- Construct the most general left-invariant (J, Ω) for each $\mathcal{M}_6$
- Calculate $(\mathrm{d}J, \mathrm{d}\Omega)$
- Impose $\tau \subset W_1^- \oplus W_2^-$

	$\mathrm{SU}(2)\times\mathrm{SU}(2)$		$\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$	$\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$	$\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$	$\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$
# of parameters	2	4	4	3	2	4
$\mathcal{W}_2^- \neq 0$	No	Yes	Yes	Yes	No	Yes
$j^6 \propto \mathrm{Re}\,\Omega$	Yes	No	Yes	Yes	Yes	No

- Topologies: $S^3 \times S^3,\ \mathrm{Tw}(\mathbb{CP}^2),\ \mathbb{CP}^3,\ S^6,\ S^5 \times S^1$

* *D. Lüst, P. Koerber & DT*, 2008

Explicit examples: cosets

- Construct the most general left-invariant (J, Ω) for each $\mathcal{M}_6$
- Calculate $(\mathrm{d}J, \mathrm{d}\Omega)$
- Impose $\tau \subset W_1^- \oplus W_2^-$
- Impose $\mathrm{d}W_2^- \propto \mathrm{Re}\Omega$; $\quad 3|W_1^-|^2 - |W_2^-|^2 \geq 0$

	$\mathrm{SU(2)}\times\mathrm{SU(2)}$	$\frac{\mathrm{SU(3)}}{\mathrm{U(1)}\times\mathrm{U(1)}}$	$\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)}\times\mathrm{U(1))}}$	$\frac{\mathrm{G_2}}{\mathrm{SU(3)}}$
# of parameters	1	3	2	1
$\mathcal{W}_2^- \neq 0$	No	Yes	Yes	No

* *D. Lüst, P. Koerber & DT*, 2008

Explicit examples: nilmanifolds

- No solutions in the absence of sources
- Two solutions with sources: on $\mathbb{T}^6$ and the Iwasawa manifold
- Related solution: $\mathbb{T}^2$ over $K3$

* *P. Koerber & DT*, 2008
* *Caviezel, Koerber, Kors, Lüst, DT & Zagermann*, 2008

Explicit examples: twistor spaces

- A two-parameter solution on

$$\mathrm{Tw}(\mathbb{CP}^2) \cong \frac{SU(3)}{U(1) \times U(1)}$$

  - misses one parameter with respect to the coset description

- A two-parameter solution on

$$\mathrm{Tw}(S^4) \cong \mathbb{CP}^3 \cong \frac{Sp(2)}{S(U(2) \times U(1))}$$

* *A. Tomasiello*, 2007
* *Feng Xu*, 2006

Explicit examples: sphere bundles

- LT vacua on $S^2(B_4)$ with B_4 positive Kähler-Einstein
  - Local *SU(2)* structure on B_4

$$\hat\omega \wedge \hat\omega^* = 2\hat\jmath \wedge \hat\jmath \; ; \quad \hat\jmath \wedge \hat\omega = 0 \; ;$$

$$\mathrm{d}\mathcal{P} = 6\, \hat\jmath \; ; \quad \mathrm{d}\hat\jmath = 0 \; ; \quad \mathrm{d}\hat\omega = i\mathcal{P} \wedge \hat\omega$$

  - Global *SU(3)* structure

$$J = |h|^2 j + \frac{i}{2} K \wedge K^* \; ; \quad \Omega = h^2 \omega \wedge K$$

where

$$j := \cos\theta \; \hat\jmath + \sin\theta \; \Re(e^{i\psi}\hat\omega)$$

$$\omega := -\sin\theta \; \hat\jmath + \cos\theta \; \Re(e^{i\psi}\hat\omega) + i \; \Im(e^{i\psi}\hat\omega)$$

$$K := f\mathrm{d}\theta + ig(\mathrm{d}\psi + \mathcal{P})$$

* *R. Terrisse & DT,* 2017

Algebraic geometry constructions
(can we eat our cake and have it too?)

- Susy selects a non-integrable complex structure on $\mathcal{M}_6$
but $\mathcal{M}_6$ may also admit another, integrable complex structure.
- Use the underlying algebra-geometric description of $\mathcal{M}_6$
Example: 3d smooth, compact toric varieties

* *M. Larfors, D. Lüst & DT*, 2010

Algebraic geometry constructions

(can we eat our cake and have it too?)

- A d-dimensional SCTV corresponds to a fan Σ
- A fan is a (certain) collection of (certain) cones generated by

$$G(\Sigma) = \{v_1, \ldots v_n\} \; ; \quad v_i \in N \cong \mathbb{Z}^d$$

- Classification of 2d SCTV
- Partial classification of (minimal) 3d SCTV
  - $\mathbb{CP}^2$ bundles over $\mathbb{CP}^1$
  - $\mathbb{CP}^1$ bundles over 2d SCTV
  - complete results for $n \leq 8$

* *Miyake & Oda; Oda,* 1978

- Some work needed to read off $G(\Sigma)$

Algebraic geometry constructions
(can we eat our cake and have it too?)

- A SCTV can also be described as a *symplectic quotient*

$$\mathcal{M}_{2d} = \mu^{-1}(0)/U(1)^s$$

- Moment maps

$$\mu^a \equiv \sum_{i=1}^{n} Q_i^a |z^i|^2 - \xi^a$$

$$a = 1, \ldots s \; ; \quad (z^1, \ldots, z^n) \in \mathbb{C}^n \; ; \quad d = n - s$$

- $U(1)^s$ action on $\mathbb{C}^n$

$$z^i \longrightarrow e^{i\varphi_a Q_i^a} z^i$$

- Unique topology for $\xi^a \in \mathcal{K}_{\mathcal{M}}$

Algebraic geometry constructions
(can we eat our cake and have it too?)

- Given $G(\Sigma)$ we determine the $U(1)^s$ charges by solving

$$\sum_{i=1}^{n} Q_i^a v_i = 0$$

- Forms Φ on $\mathbb{C}^n$ that are *gauge-invariant* and *vertical*

$$\mathcal{L}_{\mathrm{Im}V^a}\Phi = 0 \ ; \quad \iota_{V^a}\Phi = \iota_{\bar{V}^a}\Phi = 0 \quad \text{where } V^a \equiv \sum_i Q_i^a z^i \partial_{z_i}$$

descend to well-defined forms on $\mathcal{M}_{2d}$

Algebraic geometry constructions

(can we eat our cake and have it too?)

- Sufficient conditions for global *SU(3)* structure on SCTV

(1,0)-form K on $\mathbb{C}^n$ such that

1. $P(K) = K$
2. $Q^a(K) = \frac{1}{2} Q^a(\Omega_{\mathbb{C}})$
3. $|K|^2 = 2$

- Local *SU(2)* structure on SCTV

$$\omega = -\frac{i}{2} K^* \cdot \widetilde{\Omega} \; ; \quad j = \widetilde{J} - \frac{i}{2} K \wedge K^*$$

where

$$\widetilde{\Omega} \propto \prod_{a=1}^{s} \iota_{V^a} \Omega_{\mathbb{C}} \; ; \quad \widetilde{J} = P(J_{\mathbb{C}})$$

Algebraic geometry constructions

(can we eat our cake and have it too?)

- Global *SU(3)* structure on SCTV

$$J = \alpha j - \frac{i\beta^2}{2} K \wedge K^* \ ; \quad \Omega = \alpha\beta e^{i\gamma} K^* \wedge \omega$$

- Torsion classes
  - Must be computed case-by-case
  - Generally $W_i \neq 0$
  - Special points with $W_1, W_3, W_4 = 0$
  - Exception: the LT vacuum on $\mathbb{CP}^3$

Algebraic geometry constructions

(can we eat our cake and have it too?)

- Formalism applicable to non-compact toric varieties
  * *Chen, Dasgupta, Franche, Katz, Tatar,* 2010

- Many further examples of K constructed
  * *M. Larfors,* 2013

- Potentially relaxing the non-vanishing condition on K
  * *S. Dabholkar,* 2013

Algebraic geometry constructions
(can we eat our cake and have it too?)

- What about the LT vacuum on $\mathrm{Tw}(\mathbb{CP}^2)$?
- Modification of the prescription to obtain a global *SU(3)* structure on (toric) $\mathbb{CP}^1$ bundles over arbitrary 2d SCTV
  - Toric 3d *U(1)* charges

$$Q_I^A = \begin{pmatrix} q_i^a & -n^a & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad ; \quad n_a \in \mathbb{N}$$

where q_i^a are the toric 2d *U(1)* charges

  - Prescription works for $n^a = \sum_{i=1} q_i^a$
  - Generic torsion classes

✱ *R. Terrisse & DT,* 2017

Algebraic geometry constructions
(can we eat our cake and have it too?)

- *SU(3)* structures on CICY from Machine Learning

* *Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle,* 2021

proposed the *SU(3)* ansatz

$$J = \sum_{i=1}^{m} a_i J_i \ ; \qquad \Omega = A_1 \Omega_0 + A_2 \Omega_0^*$$

defined on

$$\left[\begin{array}{c|ccc} \mathbb{CP}^{n_1} & q_1^1 & \dots & q_K^1 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbb{CP}^{n_m} & q_1^m & \dots & q_K^m \end{array} \right] \ ; \qquad \sum_r q_r^i = n_i + 1$$

subject to

$$|A_1|^2 + |A_2|^2 = \sum_{i,j,k=1}^{m} \Lambda_{ijk} a_i a_j a_k$$

Algebraic geometry constructions

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* *Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle,* 2021

where the are Λ_{ijk} are read off of

$$J_i \wedge J_j \wedge J_k = \frac{3}{4} i \Lambda_{ijk} \Omega_0 \wedge \Omega_0^*$$

- The ansatz would produce LT *SU(3)* structures for

$$a_i = \text{const.}\ ; \qquad A_2 = -A_1^* + \text{const.}$$

- Unfortunately the ansatz of *Anderson etal* does not satisfy complex decomposability of Ω.

* *M. Larfors, A. Lukas, F. Ruehle & DT,* 2022

Conclusions—wishlist

- LT structures are well-motivated in supergravity/string theory
- Still few known explicit examples
- *SU(3)* decomposable ansätze on CYs and/or toric varieties ?
- Interplay between *math.AG* and *math.DG*
- Connection with Hitchin uplift to G_2. Use 7d technology?
- Existence theorems? Deformations?
- Do all LT-structure manifolds without sources admit a NK limit?

- Ground for new discoveries!